Summer School in Grenoble, June 18th-July 6th, 2007
Geometry of complex projective varieties and the minimal model program.

Expected Scientific Program (Preliminary version, November 2006).

First week. Target audience for all three courses: masters-level students.

First course by Robert LAZARSFELD. Introductory course on linear series.

All the areas appearing in the schedule are covered in the book "Positivity in Algebraic Geometry" (volumes one and two) by Lazarsfeld which is the main reference for this course. The course will be taught from an algebraic characteristic 0 standpoint : neither analytic methods nor characteristic p will be used. We suggest starting with smooth varieties only, in order to avoid frightening the beginning graduate students who constitute the target audience. Singular varieties could be introduced once course 3 (Terminal and canonical singularities) has made reasonable progress.

This course should provide the necessary background for courses 1 and 3 of the second week.

Course schedule:

(1) Definitions and vocabulary : Cartier, ample, big and nef divisors and fundamental results (Serre, asymptotic Riemann-Roch, Nakai-Moishezon, Kleiman...)
(2) Vanishing theorems. Kodaira's theorem and Kawamata-Viehweg vanishing. The proof of Kodaira's theorem is not required, as it will be given in course 2.
(3) Multiplier ideals and Nadel's vanishing theorem.
(4) Relative versions of the above.
(5) Formalism of multiplier ideals and their applications. Nadel's theorem and Kawamata's theorem for varieties with terminal singularities (terminal singularities will have been introduced concurrently in course 3).

Second course by Jean-Pierre DEMAILLY. Introduction course on Hörmander's L_2 methods.

This course will be taught from an entirely analytic standpoint. The aim will be to prove some of the theorems stated without proof in course 1.

Course schedule.

(1) Basic Hodge theory.
(2) Vanishing theorems (mainly Kodaira's vanishing theorem).

(3) Plurisubharmonic functions and singular metrics.
(4) Nadel's vanishing theorem.

Third course by Massimiliano MELLA. Terminal and canonical singularities.

Course schedule.

(1) Generalities on singularities and pluricanonical forms. Definition of terminal and canonical singularities. The surface case.
(2) Canonical singularities are the singularities of canonical models, examples of canonical models.
(3) Introduction to the classification of 3-fold terminal singularities. Examples of weighted blow ups, divisorial extractions and resolutions. Definition and examples of flips: GIT quotients of affine space and G. Brown hypersurface flips.

We will organise exercise sessions in the early afternoons and question sessions.

Second week. Target audience for all four courses: thesis students.

First course by Olivier DEBARRE. Bend and Break.

Course schedule.

Construction of rational curves. The main result of this course will be that the extremal rays of the cone theorem on terminal varieties (or alternatively log-terminal pairs) are rational curves such that $K \cdot C < 0$. Bounds will be given on $-K \cdot C$.

Second course by Massimiliano MELLA. Singularities of pairs.

This will be the sequel to the third course of the first week.

Course schedule.

(1) Motivation for working with pairs: open varieties, Kodaira formula, categorical properties and finite morphisms, Zariski-Mumford subadjunction ("Italian" definition of the canonical class), other coefficients creep up from restriction, technical usage of small coefficients e.g. to "direct" flops.
(2) Singularities of pairs: klt, plt, dlt. Basic calculation of discrepancies. Examples. A klt pair has finitely many valuations with negative discrepancy. Extensive warnings about the use of dlt singularities.
(3) Non-klt locus and centers of LC singularities. Connectedness Theorem. Perturbation arguments. Application to biregular and birational geometry.
(4) Categorical properties, inversion of adjunction.

Third course by Alessio CORTI. Introduction to the Minimal Model Program.

Course schedule.

(1) Statement of the cone theorem and the contraction theorem; the minimal model program, flip conjectures, abundance conjecture.

(2) Discussion of the $\mathbb{Q}$-factorial condition and the non $\mathbb{Q}$-factorial minimal model program.
(3) Variations. Terminal models. The minimal model program "with scaling", applications.
(4) A humane introduction to the "X-method": P. Francia's lemma on surfaces, effective base point freeness on surfaces following Ein and Lazarsfeld.
(5) Proof of the non-vanishing Theorem. Some applications of the X-method to the geometry of Fano varieties.
(6) Elementary properties of codimension 2 surgery in algebraic geometry.

Fourth course by Jaroslaw WIŚNIEWSKI. Extremal contractions.

Course schedule.

(1) Divisorial contractions. Divisorial contractions for three-dimensional smooth varieties. Other examples in dimensions 3 or 4.
(2) Mori fibre spaces. Examples (and possibly a partial classification) in dimensions 3 and 4.
(3) Small contractions. Examples (and possibly a partial classification) in dimensions 3 and 4.

We will organise exercice sessions in the early afternoons.

Third week. Target audience: graduate and post-graduate students.

First course by Alessio CORTI. Introduction to flips.

Course schedule.

(1) Reduction of klt flips to pl flips.
(2) Pl flips in dimension n as a finite generation problem in dimension n-1. The "lifting lemma" of Hacon and McKernan (statement only), discussion.
(3) Fano graded algebras and adjoint algebras. Discussion of the saturation condition. Example: curves. Saturation and the X-method.
(4) The restricted algebra is an adjoint algebra.
(5) Putting together the pieces: existence of flips.

Second course by Gianluca PACIENZA. Pluricanonical systems on algebraic varieties of general type, after Hacon-McKernan, Takayama and Tsuji.

Course schedule.

(1) The Angehrn-Siu theorem and its variant for big divisors.
(2) Lower bounds on the restricted volumes and point separation of pluricanonical systems.

Third course by Mihai PAUN. Extension of pluricanonical forms.

The speaker will present a selection of recent results in this area (invariance of plurigenera, etc) preferably using analytic methods.

Fourth course by James McKERNAN. Finite generation of the canonical ring.

Course schedule.

(1) Shokurov's polyhedral decomposition, with applications to the geometry of the moduli space of curves, and finiteness of minimal models.
(2) Termination of the minimal model program with scaling. Applications to birational geometry.
(3) Non-vanishing theorem, revisited.